

The Vigesimal Numeral System: Another Logic for Constructing and Using Amounts in the Maya Culture's Community Practices

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This study is situated in the Maya region of the state of Chiapas, in southern Mexico. The schools in the region do not take into account the numerical knowledge that students acquire in their community. Children arrive at school with mathematical skills based on the vigesimal numeral system. This ethnographic study attempts to discover the logic of the vigesimal numeral system in community practices, including buying and selling at the public market, planting and harvesting corn and coffee, and producing embroidered items. The historical and cultural perspective [22] is a basic reference for analysis in the study. While some studies focus on mathematical procedures within cultures [21; 5; 18; 7], and others center on the Maya culture's written expression [19; 12; 1; 11], no previous studies have looked at mathematical knowledge based on oral elements. The results of analysis show that the vigesimal numeral system has a specific logic that is evident in various aspects, such as the construction of names of amounts, commercial transactions, and the women's calculations for embroidery work. A unique aspect of this logic is the relation the Maya people make between the human body and their names for numbers. Also unique in the Maya's use of the vigesimal numeral system is the way they construct phrases while buying and selling products in the market: they use both languages and two different numeral systems to construct a single phrase.

Keywords: Numeral system; community practices; culture; oral.

Introduction

This research was completed in the context of a specific culture: the Maya-Tzeltal culture in the state of Chiapas, Mexico. The communities participating in the study are located in the municipalities of Chilon and Ocosingo.

In Western culture, mathematics have been associated generally with abstract aspects. There are some beliefs in some communities in Mexico relative to the people constructed the bases of mathematics, including numeral systems, during their free time.

If we review the history of mathematics, however, we discover that cultures' use of numbers and their construction of the numeral system have been based on human needs. For example, the Greek culture had spatial concerns related to the polis that led to the precise development of geometry.

The intent of this study is not to address the cultural origins of numbers, but to emphasize the importance of considering the aspects of culture and context that influence numerical concepts in the Maya culture and its communities.

The study is based on ethnographic theory [15]. The analyses presented below are a result of fieldwork carried

out in 2007 and 2008 with young people, adults, and elders in Tzeltal communities. Some data are from taped interviews and other data are part of field notes logs.

Statement of problem

Schools show similar situations when teaching numbers and the Spanish language versus the native language. They do not take into account the numerical knowledge that children have learned in their native culture (counting, and the names of numbers), and begin to teach them the decimal system. For example, schools do not consider that children from the Tzeltal culture use a vigesimal numeral system.

The lack of integration between Maya students' previous mathematical knowledge and the strategies implemented in the mathematics classroom may be one of the causes for student difficulties. Therefore, we decided to focus this research on the knowledge and current use of the vigesimal numeral system in the Maya culture.

We started with the premise that mathematical knowledge is a cultural and historical construction, in addition to an issue of social equality [3]. Through

mathematics, children develop skills that contribute to the quality of life in diverse settings. All of Mexico's youth should have equal access to the opportunity to learn these skills. Saxe reflects the concerns that provide the basis for this study:

«One problem across studies of schooling in traditional cultures is that researchers have not typically investigated the role of knowledge forms indigenous to children's culture in guiding children's interaction with the novel school curriculum. This neglect has been manifest in a disregard for the characteristics of indigenous cognitive forms, the use of dependent measures developed outside the settings of the group under study, and failure to consider the possible interplay between indigenous knowledge forms and Western school concepts in discussing the character of school effects» [17, p. 503–504].

Taking the above into account, we can infer the need to study the use of the vigesimal numeral system in the daily practices of Indian communities.

Historical and Cultural Context

During the construction of Mexico as a modern nation, the voice of the native peoples (Indians) has been excluded. The nation's social structure has been developed by mestizos, the mixed-race descendants of the Spanish and Indians. In 1521, the Spanish conquest decimated the native population. Three centuries later, Mexico declared its independence. In this process, the voice of the native peoples was not included [(Hernandez, 1998). At the present time, 12 % of the population (112 million inhabitants) can be classified as native people. In the indigenous communities, the public schools have implemented educational models with a Spanish-speaking focus, aimed at having Indians assimilate into the mestizo «civilization». These models generally attach more value to Western culture than to native culture.

The state of Chiapas is located in southeastern Mexico. Of a population of three million, 34 % is indigenous and mostly Maya. 17,8 % of the state's population lives in extreme poverty. Chiapas is one the nation's wealthiest states, however, in natural resources: jungles, forests, beaches, waterfalls, lagoons, and rivers. In the educational sphere, Chiapas has the lowest indexes of educational quality in Mexico (high dropout rates, high failure rates, and high absentee rates). Of the population between ages six and fourteen, 26,94 % speak an indigenous language. The number of incomplete indigenous schools (schools that have only a few grades of basic elementary education) is 863 out of a total of 2452, or 35,20 % [8]. Of the thirteen Maya groups, the five largest are Tzeltal, Totsil, Tojolabal, Ch'ol, and Mam. They live in the mountain and jungle regions of Chiapas. The Tzeltal occupy primarily the border zone between the mountains and jungle, and most of them live in the topical canyons; they represent 10 % of the state's total population, with 392,000 inhabitants. Their principal source of work is growing coffee, corn, beans, sheep, and goats, and producing honey and handicrafts (De Vos,

2008). The cultural references of the Tzeltal groups are linked to a cosmic view based on contact with nature. The oldest expression of these references is the book of *Popol Vuh*.

The community organization consists of elected authorities. Some positions, like teaching, involve service. Teachers are called *nopteswanej* in the Mayan language, literally «he who helps in learning». The elders are recognized as sources of wisdom and are known as «principals». They participate mainly in religious rituals and work to restore community harmony in situations of conflict [13].

Logic of constructing numbers in the maya-tzeltal community

Geertz quoted Max Weber's image of humanity as «animals suspended in nets of meaning that they have woven themselves». He added, «I believe that those nets are culture» [6, p. 118].

In this sense, we have discovered that mathematical constructions are related to daily life, in a framework of culture as well as context. Context is defined by Michael Cole [6] as «that which interlinks»; context is also understood as «the connected whole that gives coherence to its parts». In the paragraphs below, we shall attempt to present the specific aspects of numbers in Maya community practices as related to the whole of Maya culture.

In the past, the Maya's primary interest was time [11]. Their numerical construction was linked to the period or date for planting, harvesting, or selling; to religious rituals; and to astronomical observation. The manuscript that describes these interests is the Dresden Codex, one of the few codices saved from burning during the Spanish conquest [12].

A characteristic of the Maya's numeral system is that two numbers are fundamental in mathematical constructions: 20 and 13. Each number has a specific function. Both relate to the human body and represent its totality: 20 is the number of a person's fingers and toes, and 13 is the total number of primary articulations. These numbers are the basis of a wide variety of aspects within Maya culture.

The Number 20

The number 20 represents a man who has ten fingers and ten toes, as explained by a community elder:

«Because those are the old customs of our ancestors. Because they did not know how to count in Spanish. They didn't have a school and they were used to talking in Tzeltal. But they thought through their hearts and they began to count the numbers 1, 2, 3, 4, 5... on their fingers, but in Tzeltal. Not only with their fingers, but also with their toes: *jun, cheb, oxeb, chaneb, vo'eb* until reaching the number 20, which is *jun winik (tab)* in Tzeltal. That's how they counted».

The Number 13

The number 13 represents the number of primary articulations in the human body: shoulders (2), elbows (2), wrists (2), hips (2), knees (2), ankles (2), and neck (1).

The following account shows the way 13 is used in various dimensions of daily life:

«On one occasion, one of the teachers arrived in the community with an eye problem. It was swollen and red, with an infection, and the teacher called it a “sty”. Then the lady of the house where the teacher was staying told him: “To get rid of your sickness, you take 13 kernels of corn to rub on your eyes where you have the inflammation (sty), one after another”. The teacher asked her, «How many times do I rub each kernel on my eye? 13 times. He did it like that and he was cured».

Home remedies are prescribed in periods of 13 days.

The Numbers 20 and 13

The combination of the two amounts, 20 and 13, is the origin of the prophetic calendar known as Tzolkin ($20 \times 13 = 260$), meaning 260 days [1; 11; 12].

The combined prophetic cycle was obtained by multiplying 20×13 . The number 20 corresponded to the designated gods according to the particular characteristics of the days [11]. In addition, the number 20 refers to the human body's total of 20 fingers and toes [10]. The number 13 was a number or figure that some believe to be only a number [11]; based on the data collected from today's Tzeltal communities, we know that it corresponds to the number of primary joints in the human body. Thus we can appreciate the Maya's references to the human body for constructing amounts and designating cycles.

Naming Amounts in Tzeltal

The reference to the human body is fundamental for naming amounts in Maya-Tzeltal. The basic unit considers a man's fingers and toes: 20. The name of the number 20 is *jun winik*, which means one man. As mentioned above, the Maya elders' stories tell the way their ancestors counted with their fingers. This observation defined 20 as the basis of their numeral system.

Naming amounts follows a corporal logic of complete men (groups of 20) implied by a given amount, called prospective [1]. Amounts are named by saying the units that remain after having identified the number of complete men. Thus we say that numbers are based on the last «fingers and toes» or units that remain after considering complete men. The remaining fingers and toes are mentioned and then the name of the man to which they belong – the man following the previous complete man.

For example, to say the number 46, first the number 6 is pronounced and then the number 60 (*wakeb oxwinik*). Why is 6 (*wakeb*) related to the number sixty (*oxwinik*)? The logic is as follows: The number 46 contains two complete groups of 20 (complete men) who

total 40. The six remaining units do not pertain to either of the first two groups of 20, but to the third group of 20 (third complete man) – the number 60. In summary, the remaining units are named once the number of complete men has been identified, followed by the name of the man to which the remaining units pertain.

To name a larger amount, calculations identify the number of complete men in the amount, plus the remainder, as shown by the following dialogue with a young man:

Researcher: Yes, that's it. Now, for example, this number: 199. How would you say it?

H: 199 would be..... (10 seconds) *balun... balunlajuneb lajunwinik... lajunwinik*.

Researcher: How do you know? How do you do it to know it is *balunlajuneb lajunwinik*?

H: It would be 19 units of the tenth man.

Researcher: Oh, I see.

H: It's almost, almost ten men. One unit is missing to get to ten men.

Researcher: What do you think about first when you see this number?

H: What do I think about first? Fast I go 20 by 20 because ten men would be 200, and 199 would be *balunlajuneb lajunwinik*. *Balunlajuneb lajunwinik*.

In this case, the young man first relates the amount, 199, to the closest number of complete men, which is 200, realizing that 10 complete men (10 men with 20 fingers and toes) is equal to 200 fingers and toes.

Following our Western logic, we can state that 9 groups of 20 (180 units) fit in 199, and that 199 minus 180 gives us 19 units, and that these 19 units belong to the tenth man. Therefore, the name must be 19 (*balunlajuneb*) of the tenth man (*lajunwinik*). In Tzeltal, one would say *balunlajuneb lajunwinik*.

Logic of daily use of numbers in the maya-tzeltal community

Units of Measurement of the Maya-Tzeltal Culture

In some cases, the measurement used is well determined. For example, in planting and harvesting corn, *zonte* establishes the number corresponding to 200 ears of corn. The number of ears that fit in a bag is stipulated: «200 ears in a bag».

The following dialogue between the interviewer and a young man involves the corn harvest.

Researcher: When a man is going to pick corn, he gets 350 ears in a day. How many bags or nets will he need to take the ears home?

E: Two bags or two nets.

M: Why two bags or nets?

E: Because 200 ears fit in each bag, and there is still room for 50 more.

The participant used his previous cultural knowledge (a bag has 200 ears), in relation to the base 20 of the vigesimal numeral system [17] as well as base 10.

Indications of the relation between the proposed arithmetic problem and the vigesimal numeral system can be seen in the above example. The student searches for a balance between the total and the number of parts with a culturally defined unit [7]. Amounts are grouped around 200, which is a multiple of 20. The young man knows the number of ears that fit in a bag, and compares that unit with the total ears picked. He finds not only the result – two bags – but also the amount that can still fit (50) in the two bags (400).

Ways of Counting Based on Embroidery Experience

Another way that community practices relate to numbers is manifest in the Maya's description of completing a task (with implied adjustments in proportion) as «working with the heart». Embroiderers use this expression to explain how they embroider small items instead of large items like blouses. They are referring specifically to the size of the embroidered figures.

In this section, we shall attempt to understand this relationship by asking the following questions: What does «working with the heart» mean for the Maya? What is the heart's place in the Maya culture? What are the implications of describing an imminently numerical task as «working with the heart»? What are the implied concepts and forms of perception? How does this form of relation refer to a unique way of using mathematics?

Embroiderers work with garments (mainly blouses), household items (mats), and items for carrying objects (bags). They sew and embroider items of various sizes. In the case of blouses, they produce more large blouses for women than small blouses for girls. If we ask the embroiderers how they calculate the size of the embroidered figure on a girl's blouse, they answer that they «work from the heart». The same reply is used to explain how they calculate a smaller mat than usual. Producing a smaller figure based on a larger figure is a topic of proportions.

In Western mathematics, a proportion is equality between two ratios, such as: $a/b = c/d$. For example, $2/5 = 4/10$, indicating that the two amounts that are related on an object can be represented on another object in lesser or greater amounts, provided the ratio between the two is respected [14].

Using Western mathematics, we can establish that proportions are a fundamental topic for defining the relation between a large and small blouse. However, rather than indicating a direct numeric activity, the embroiderers' expressions have to do with a fundamental aspect of the Maya's cosmic vision: the heart.

The Maya's cosmic vision associates the mind and the heart. For example, the statement is made that «the mind is the head of the heart». For example, «If someone is forgetful, his heart does not take care of things. Whatever your eyes see, goes does to your heart and then rises to your thinking. Your heart is like a treasure chest and it takes care of everything you see» [9].

Thoughts are in your head. «Let's think with the heart's head». Thinking is the heart's head. «What we see goes to our heart, and from there to our head, because the heart is the mother of our blood, and then it passes through the air» [9].

The heart in the Maya's cosmic vision «keeps all wisdom and is the seat of the memory and knowledge. Through the heart, perception occurs, and emotions are an aid for mental processes, as well as for the body's functions».

If we turn to Vygotsky's historical and cultural focus in psychology, which states that knowledge is constructed through socialization with adults from a determined culture upon carrying out shared social activities, we discover that the concept of activity is crucial. From the viewpoint of cultural psychology, activity has its own meaning. Activity is not a type of action, but is *social, practical, and shared*. This concept contains symbolic exchange and the use of cultural tools for mediation. In such activity, therefore, adults and non-adults, and expert and novice individuals meet. Meaning is created in activity, and practical, emotional, relational, and cognitive aspects are integrated.

The embroiderers, for example, carry out a joint activity that employs cultural constructs whose meanings are shared by the community's members. The meaning under study refers not only to the heart as a physical organ or to the heart's symbolic aspect in emotions, but also to the rational aspect of the heart: the heart's function in realizing numerical operations.

At the market, buying and selling are joint activities that are practical and specific. They employ a mixture of languages and numerical references that show that culture is a dynamic construct that is transformed by shared activities.

Bruner suggests that «The most general implication is that a culture is constantly re-created by being interpreted and renegotiated by its members». According to this perspective, culture is a place for negotiating and renegotiating meanings and explaining action, like a set of rules or specifications for action [4, p. 128].

We shall see how the names of numbers for products and money are used in market transactions in the Tzeltal region:

Ways of Buying and Selling: Two Languages and Two Numeral Systems in a Single Phrase

Sometimes the Tzeltal use numbers in both languages – Tzeltal and Spanish – to say the price of a product and the amount of product that corresponds to that price. The amount of the product is stated in Tzeltal and the price is stated in Spanish. The amounts in Tzeltal use the vigesimal numeral system, while the amounts in Spanish use the decimal numeral system.

Two amounts, in two languages, can be used in a single phrase to refer to two numeral systems. For example, *chaneb plátano ta cinco pesos*, means that four

bananas cost five pesos. We may wonder about the cultural implications of using numbers in this manner. The amount said in Spanish, in the decimal numeral system, is tied to a specific magnitude: money. Money is associated with the Mexican monetary system, based on the decimal numeral system: bills and coins and a unit of one peso. A monetary system is the basis for defining the type of objects represented by money and value. The Tzeltal use the monetary system stipulated by the government of the country where they live: Mexico.

When the Tzeltal use Spanish to refer to the price, they are using the monetary system stipulated by the nation's «dominant» culture. This monetary system's influence is shown by the use of the number in Spanish to specify the amount of money. This influence, however, does not affect the amounts of products bought or sold (*chaneb plátano*), which are named in Tzeltal in relation to the vigesimal numeral system. Thus we can confirm a limit on the influence of the decimal numeral system in the Maya culture, which continues using the

Tzeltal language for counting objects unrelated to money.

Conclusions

The logic of construction and the use of numbers in the Tzeltal community revolve around multiples of 20 and 13. Particularly in daily practices, the number 20 is the base for constructing measures through the solution of everyday problems. We discovered that the use of numbers is related to specific aspects of the Maya's cosmic vision that would appear to be separate from numbers; research on the origins and use of certain expressions revealed numerical topics like proportion. Lastly, we found that the uses of numbers combine with numerical knowledge from the outside culture: some phrases feature two languages and two numeral systems. The diversity of usage shows the dynamism of this culture, conceived as a changing pattern of networks of meaning that are updated by community practices.

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Двадцатеричная система счисления: Другая логика для построения и использования величин в общественных практиках культуры Майя

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Это исследование проводится в регионе Майя в штате Чьяпас на юге Мексики. Школы в данном регионе не принимают во внимание численные знания, которые студенты приобретают в своей общине. Дети приходят в школу с математическими навыками, основанными на двадцатеричной системе счисления. Данное этнографическое исследование стремится раскрыть логику двадцатеричной системы счисления в общественных практиках, в таких как покупка и продажа на общественном рынке, посадка и уборка урожая кукурузы и кофе, производство предметов с вышивкой. Исторический и культурный подход [22] является основным при осуществлении анализа в исследовании. Хотя некоторые исследования посвящены математическим процедурам в различных культурах [21; 5; 18; 7], а другие фокусируются на письме в культуре Майя [19; 12; 1; 11], но математические знания на основе устных элементов ранее не рассматривались. Результаты анализа показывают, что двадцатеричная система счисления имеет специальную логику, которая проявляется в различных аспектах, таких как конструирование названий величин, коммерческих операциях и в расчетах женщин для работ по вышиванию. Уникальной особенностью этой логики являются соотношения, которые майя производят между телом человека и названиями чисел. Также уникальным в использовании майя двадцатеричной системы счисления является способ, которым они строят фразы при покупке и продаже товаров на рынке: они используют оба языка и две различные системы счисления при построении одной фразы.

Ключевые слова: система счисления, общественные практики, культура, оральный.